11-3 Simplifying Rational Expressions

State the excluded values for each rational expression.

1. \( \frac{8}{x^2 - 16} \)
   ANSWER: 4, -4

2. \( \frac{3m}{m^2 - 6m + 5} \)
   ANSWER: 1, 5

3. PHYSICAL SCIENCE  A 0.16-kilogram ball attached to a string is being spun in a circle 7.26 meters per second. The expression \( \frac{mv^2}{r} \), where \( m \) is the mass of the ball, \( v \) is the velocity, and \( r \) is the radius, can be used to find the force that keeps the ball spinning in a circle. If the circle has a radius of 0.5 meter, find the force that must be exerted to keep the ball spinning. Round to the nearest tenth.
   ANSWER: 16.9 units of force

Simplify each expression. State the excluded values of the variables.

4. \( \frac{28ab^3}{16a^2b} \)
   ANSWER: \( 7b^2 \); \( a \neq 0, b \neq 0 \)

5. \( \frac{(-3r)(10r^4)}{6r^5} \)
   ANSWER: -5; 0

6. \( \frac{5d + 15}{d^2 - d - 12} \)
   ANSWER: \( \frac{5}{d-4} \); 4, -3

7. \( \frac{x^2 + 1lx + 28}{x + 4} \)
   ANSWER: \( x + 7; -4 \)

8. \( \frac{2r - 12}{r^2 - 36} \)
   ANSWER: \( \frac{2}{r + 6}; 6, -6 \)

9. \( \frac{3y - 27}{81 - y^2} \)
   ANSWER: \( \frac{-3}{9 + y}; -9, 9 \)

Find the zeros of each function.

10. \( f(x) = \frac{x^2 - x - 12}{x - 2} \)
    ANSWER: 4, -3

11. \( f(x) = \frac{x^2 - x - 6}{x^2 + 8x + 12} \)
    ANSWER: 3

State the excluded values for each rational expression.

12. \( \frac{-n}{n^2 - 49} \)
    ANSWER: 7, -7

13. \( \frac{5x + 1}{x^2 - 1} \)
    ANSWER: 1, -1

eSolutions Manual - Powered by Cognero
11-3 Simplifying Rational Expressions

14. \( \frac{12a}{a^2 - 3a - 10} \)
   
   ANSWER: 5, -2

15. \( \frac{k^2 - 4}{k^2 + 5k - 24} \)
   
   ANSWER: 3, -8

16. GEOMETRY The volume of a rectangular prism is \(3x^3 + 34x^2 + 72x - 64\). If the height is \(x + 4\), what is the area of the base of the prism?
   
   ANSWER: 
   
   \(3x^2 + 22x - 16\)

17. GEOMETRY Use the circle to write the ratio \(\frac{\text{circumference}}{\text{area}}\). Then simplify. State the excluded value of the variable.

   \[ \frac{2\pi(5t)}{\pi(5t)^2} = \frac{2}{5t}; 0 \]

   ANSWER: 
   
   Simplify each expression. State the excluded values of the variables.

18. \( \frac{15x^4y^2}{40x^3y^3} \)
   
   ANSWER: \(\frac{3x}{8y}; x \neq 0, y \neq 0\)

19. \( \frac{32n^2p}{2n^3p} \)
   
   ANSWER: \(\frac{16}{n}; n \neq 0, p \neq 0\)

20. \( \frac{(4r^3)(2r)}{20r^2} \)
   
   ANSWER: \(\frac{2r^2}{5}; 0\)

21. \( \frac{(7c^3)(-6c^3)}{2kc^4} \)
   
   ANSWER: \(-2c; 0\)

22. \( \frac{4x - 24}{x^2 - 12x + 36} \)
   
   ANSWER: \(\frac{4}{x - 6}; 6\)

23. \( \frac{a^2 + 3a}{a^2 - 3a - 18} \)
   
   ANSWER: \(\frac{a}{a - 6}; 6, -3\)

24. \( \frac{n^2 + 7n - 18}{n - 2} \)
   
   ANSWER: \(n + 9; 2\)

25. \( \frac{x^2 + 4x - 32}{x + 8} \)
   
   ANSWER: \(x - 4; -8\)

26. \( \frac{x^2 - 25}{x^2 + 5x} \)
   
   ANSWER: \(\frac{x - 5}{x}; 0, -5\)
11-3 Simplifying Rational Expressions

27. \( \frac{2p^2 - 14p}{p^2 - 49} \)
   \( \text{ANSWER:} \quad \frac{2p}{p+7}; 7, -7 \)

28. \( \frac{2x-10}{25-x^2} \)
   \( \text{ANSWER:} \quad \frac{2}{-(x+5)}; 5, -5 \)

29. \( \frac{64-c^2}{c^2-7c-8} \)
   \( \text{ANSWER:} \quad \frac{-8+c}{c+1}; 8, -1 \)

Find the zeros of each function.

30. \( f(x) = \frac{x^2-x-12}{x^2+2x-35} \)
   \( \text{ANSWER:} \quad -3, 4 \)

31. \( f(x) = \frac{x^2+3x-4}{x^2+9x+20} \)
   \( \text{ANSWER:} \quad 1 \)

32. \( f(x) = \frac{2x^2+11x-40}{2x+5} \)
   \( \text{ANSWER:} \quad -8, \frac{5}{2} \)

33. \( f(x) = \frac{3x^2-18x+24}{x-6} \)
   \( \text{ANSWER:} \quad 2, 4 \)

34. \( f(x) = \frac{x^3+x^2-6x}{x-1} \)
   \( \text{ANSWER:} \quad 0, -3, 2 \)

35. \( f(x) = \frac{x^3-4x^2-12x}{x+2} \)
   \( \text{ANSWER:} \quad 0, 6 \)

36. **PYRAMIDS** The perimeter of the base of the Pyramid of the Sun is 4\( \pi \) times the height. The perimeter of the base of the Great Pyramid of Giza is 2\( \pi \) times the height. Write and simplify each ratio comparing the base perimeters.

<table>
<thead>
<tr>
<th>Pyramid</th>
<th>Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyramid of the Sun</td>
<td>233.5</td>
</tr>
<tr>
<td>(Mexico)</td>
<td></td>
</tr>
<tr>
<td>Great Pyramid</td>
<td>481.4</td>
</tr>
<tr>
<td>(Egypt)</td>
<td></td>
</tr>
</tbody>
</table>

a. Pyramid of the Sun to the Great Pyramid
b. Great Pyramid to the Pyramid of the Sun

\( \text{ANSWER:} \quad a. \) about 0.97  
\( b. \) about 1.03

37. **CCSS REASONING** George Ferris built the first Ferris wheel for the World’s Columbian Exposition in 1893. It had a diameter of 250 feet.

a. To find the speed traveled by a car located on the wheel, you can find the circumference of a circle and divide by the time it takes for one rotation. Write a rational expression for the speed of a car rotating in time \( t \).

b. Suppose the first Ferris wheel rotated once every 5 minutes. What was the speed of a car on the circumference in feet per minute?

\( \text{ANSWER:} \quad a. \quad \frac{250\pi}{t} \)

b. about 157 ft/min
11-3 Simplifying Rational Expressions

Simplify each expression. State the excluded values of the variables.

38. \( \frac{3a^2b^4 + 9a^3b - 6a^4b}{3a^2b} \)

ANSWER: 
\( b^3 + 3a - 2a^3; a, b \neq 0 \)

39. \( \frac{8x^4 - 10xy^2}{2xy^3} \)

ANSWER: 
\( \frac{4x^4 - 5y^2}{y^3}; x, y \neq 0 \)

40. \( \frac{x + 5}{3x^2 + 14x - 5} \)

ANSWER: 
\( \frac{1}{3x - 1}; x \neq \frac{5}{3}, x \neq \frac{1}{3} \)

41. PACKAGING To minimize packaging expenses, a company uses packages that have the least surface area to volume ratio. For each figure, write a ratio comparing the surface area to the volume. Then simplify. State the excluded values of the variables.

a. 

\[ \text{ ANSWER: } \frac{10x^2}{\pi r^3} = \frac{5}{x}; 0 \]

b. 

\[ \frac{2\pi a^2 + 2\pi ab}{\pi a^2b} = \frac{2a + 2b}{ab}; 0, 0 \]

42. HISTORY The diagram shows how a lever may have been used to move blocks.

a. The mechanical advantage of a lever is \( \frac{L_A}{L_R} \), where \( L_A \) is the length of the effort arm and \( L_R \) is the length of the resistance arm. Find the mechanical advantage of the lever shown.

b. The force placed on the rock is the product of the mechanical advantage and the force applied to the end of the lever. If the Egyptian worker can apply a force of 180 pounds, what is the greatest weight he can lift with the lever?

c. To lift a 535-pound rock using a 7-foot lever with the fulcrum 2 feet from the rock, how much force will have to be used?

ANSWER:

a. 4
b. 720 lb
c. 214 lb
11-3 Simplifying Rational Expressions

43. **ERROR ANALYSIS**  Colleen and Sanson simplified \( \frac{12x+36}{x^2-x-12} \) and found the excluded value (s). Is either of them correct? Explain.

- **Colleen**
  \[
  \frac{12x+36}{x^2-x-12} = \frac{12(x+3)}{(x-4)(x+3)}
  \]
  The excluded values are 4 and -3.

- **Sanson**
  \[
  \frac{12x+36}{x^2-x-12} = \frac{12(x+3)}{(x-4)(x+3)}
  \]
  \[
  = \frac{12}{x-4}
  \]
  The excluded value is 4.

**ANSWER:**
No; Colleen did not show the simplified expression, and Sanson used the simplified expression to find the excluded values.

44. **CCSS PRECISION**  Compare and contrast the key features of the graphs of \( y = x - 2 \) and \( y = \frac{x^3 + 5x - 14}{x + 7} \).

**ANSWER:**
Sample answer: The graphs appear to be identical, but the second graph has an excluded value at \( x = -7 \), so there is a hole in the graph at \((-7, -9)\).

45. **REASONING**  Explain why every polynomial is also a rational expression.

**ANSWER:**
Every polynomial \( P \) can be written as \( \frac{P}{1} \), where the numerator and denominator are polynomials; hence every polynomial is also a rational expression.

46. **OPEN ENDED**  Write a rational expression with excluded values -2 and 2. Explain how you found the expression.

**ANSWER:**
Sample answer: \( \frac{1}{x^2 - 4} \); since the excluded values are 2 and -2, the denominator of the rational expression must contain the factors \( x - 2 \) and \( x + 2 \).

47. **REASONING**  Is \( \frac{2x^2 - 4x}{x - 2} \) in simplest form?

**ANSWER:**
Justify your answer.

No; the numerator and denominator have \( x - 2 \) as a common factor.

48. **WRITING IN MATH**  List the steps you would use to simplify \( \frac{x^2 + x - 20}{x + 5} \). State the excluded value.

**ANSWER:**
Sample answer: Factor the numerator as \((x + 5)(x - 4)\). Then divide the numerator and the denominator by the GCF, \( x + 5 \). The simplified expression is \( x - 4 \) and the excluded value is \( x = -5 \).

49. Simplify \( \frac{2x + 4}{2} \).
   A \( x + 1 \)
   B \( x \)
   C \( x + 2 \)
   D \( \frac{x}{2} \)

**ANSWER:**
C
50. SHORT RESPONSE Shiro is buying a car for $5800. He can pay the full amount in cash, or he can pay $1000 down and $230 a month for 24 months. How much more would he pay for the car on the second plan?

ANSWER: $720

51. GEOMETRY What is the name of the figure?

F triangular pyramid
G triangular prism
H rectangular prism
J triangulon

ANSWER: G

52. A rectangle has a length of 10 inches and a width of 5 inches. Another rectangle has the same area as the first rectangle but its width is 2 inches. Find the length of the second rectangle.

A 30 in.
B 60 in.
C 20 in.
D 25 in.

ANSWER: D

53. State the excluded value for each function.

53. \( y = \frac{6}{x} \)

ANSWER: 0

54. \( y = \frac{2}{x - 5} \)

ANSWER: 5

55. \( y = \frac{x - 4}{x - 3} \)

ANSWER: 3

56. \( y = \frac{3x}{2x + 6} \)

ANSWER: -3

Solve. Assume that \( y \) varies inversely as \( x \).

57. If \( y = 10 \) when \( x = 4 \), find \( x \) when \( y = 2 \).

ANSWER: 20

58. If \( y = 12 \) when \( x = 3 \), find \( x \) when \( y = 6 \).

ANSWER: 6

59. If \( y = -5 \) when \( x = 3 \), find \( x \) when \( y = -3 \).

ANSWER: 5

60. If \( y = 21 \) when \( x = -6 \), find \( x \) when \( y = 7 \).

ANSWER: -18

61. CRAFTS Melinda is working on a quilt using the pattern shown. She has several triangular pieces of material with two sides that measure 6 inches. If these pieces are similar to the pattern shown, what is the length of the third side?

ANSWER: 8.4 in.

62. \( \sqrt{20} \)

ANSWER: \( 2\sqrt{5} \)
11-3 Simplifying Rational Expressions

63. \( \sqrt{18} \)
   
   ANSWER: \( 3\sqrt{2} \)

64. \( \sqrt{2} \cdot \sqrt{8} \)
   
   ANSWER: \( 4 \)

65. \( 2\sqrt{32} \)
   
   ANSWER: \( 8\sqrt{2} \)

66. \( \sqrt{5} \cdot \sqrt{6} \)
   
   ANSWER: \( \sqrt{30} \)

67. \( \sqrt{40a^2} \)
   
   ANSWER: \( 2|a|\sqrt{10} \)

68. \( \sqrt{\frac{t}{8}} \)
   
   ANSWER: \( \frac{\sqrt{2t}}{4} \)

69. \( \sqrt{\frac{2}{7}} \cdot \sqrt{\frac{7}{3}} \)
   
   ANSWER: \( \frac{\sqrt{6}}{3} \)

70. INVESTMENTS Determine the amount of an investment if $250 is invested at an interest rate of 7.3% compounded quarterly for 40 years.

   ANSWER: about $4514.89

Find the greatest common factor for each set of monomials.

71. \( 2x, 8x^2 \)
   
   ANSWER: \( 2x \)

72. \( 3y^2, 7y^3 \)
   
   ANSWER: \( y^2 \)

73. \( 7g, 10h \)
   
   ANSWER: \( 1 \)

74. \( 21c^2d^3, 14cd^2 \)
   
   ANSWER: \( 7cd^2 \)

75. \( 9qt^2, 18q^2t^2, 27qt \)
   
   ANSWER: \( 9qt \)

76. \( 10ab, 25a^2b^2, 30a^2b \)
   
   ANSWER: \( 5ab \)